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For, bisecting  $FE$  in  $G$  and drawing  $AG$ ,  $CD = FE = 2AB$  according to the nature of the conchoid. Since  $FG = GE = AG = AB$ , we easily find  $\angle CBE = \frac{1}{3}\angle ABC$ . By the lower conchoid  $C'E'$ , the obtuse adjacent angle is trisected. For, bisecting  $E'F'$  ( $= C'D = 2AB$ ) in  $G'$  and drawing  $AG'$ , we have  $E'G' = G'A = G'F' = AB$ , from which at once follows  $\angle E'BC' = \frac{1}{3}\angle ABC'$ .

*Nicomedes* devised the conchoid for the trisection of an angle and the duplication of a cube.

3. Let  $BEC$  represent an Archimedean Spiral. Divide the radius  $BC$  of the circular arc  $AC$ , into three equal parts so that  $BD = \frac{1}{3}BC$ , then, describing from  $B$ , with radius  $BD$ , an arc which intersects the spiral at  $E$ , the angle  $ABE = \frac{1}{3}$  angle  $ABC$ . For, according to the definition of the spiral,  $AB : BE (= BD) = \angle ABC : \angle ABE$ .

4. *Montucla* ascribes the following two solutions to the Platonic school.

1. Let  $ACB$  be the angle to be trisected. From  $C$ , with any radius, describe a semi-circle, and through  $B$  draw  $BE$ , intersecting the circle in  $D$ , so as to make  $DE =$  the radius of the circle, then angle at  $E = \frac{1}{3}ACB$ .

2. Let  $ABC$  be the angle to be trisected. Complete the rectangle above  $BC$ . Produce the upper side, and through  $B$  draw  $BE$  meeting the upper side produced in  $E$  and intersecting the perpendicular  $CA$  in  $D$ , so as to make  $DE = 2AB$ , then angle  $DBC = \frac{1}{3}ABC$ , as can be easily proved by drawing  $AG$  to the middle point of  $DE$ .

5. The jesuit *Thomas Ceva* devised an instrument for the trisection of an angle. It consists of four rulers,  $AE, AF, DB, DC$ , which form a rhombus,  $ABDC$ , and are movable around  $A, B, C, D$ . (The points  $B, G$  and  $C, H$  being, respectively, on  $AE$  and  $AF$ .) If the angle  $GDH$  is to be trisected, we take  $GD = DH = BD$ , fasten the instrument at  $D$ , and move the rulers so as to make  $AE$  and  $AF$  pass through  $G$  and  $H$ , then angle  $EAF = \frac{1}{3}GDH$ .

6. By approximation we can trisect the angle  $BCA = \alpha$ , in the following manner:

Make  $AD = \frac{1}{4}\alpha$ ,  $DE = \frac{1}{4}AD = \frac{1}{4^2}\alpha$ ,  $EF = \frac{1}{4}DE = \frac{1}{4^3}\alpha$ , &c.; then we obtain for the sum of all these arcs, by summing the infinite geometric series  $(\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots)\alpha = \frac{1}{3}\alpha$ .

NOTE ON THE CATENARY, BY PROF. W. W. JOHNSON.—The following formulæ arise in the consideration of the measurement of a base line by means of a steel tape which is allowed to assume the form of a catenary.

The equation of the curve being

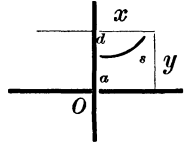
$$y = \frac{a}{2} \left( e^{\frac{x}{a}} + e^{-\frac{x}{a}} \right), \quad (1)$$

we have

$$s = \frac{a}{2} \left( e^{\frac{x}{a}} - e^{-\frac{x}{a}} \right), \quad (2)$$

whence

$$s^2 = y^2 - a^2 = (y-a)(y+a). \quad (3)$$



Now suppose the ends of the tape to be on a level, and that  $d$ , the deflection of the curve, is measured; the length of the tape is  $2s$ , and  $s$  and  $d$  are the known quantities. But  $y-a=d$ , hence from (3),  $y+a=s^2 \div d$ ; whence

$$y = \frac{s^2 + d^2}{2d}, \quad \text{and} \quad a = \frac{s^2 - d^2}{2d}. \quad (4)$$

From (1) and (2), 
$$e^{\frac{x}{a}} = \frac{y+s}{a},$$

and substituting from (4),

$$e^{\frac{x}{a}} = \frac{s^2 + d^2 + 2ds}{s^2 - d^2} = \frac{s+d}{s-d};$$

whence 
$$x = a \log \frac{s+d}{s-d} = \frac{s^2 - d^2}{2d} \log \frac{s+d}{s-d}, \quad (5)$$

or, putting  $L$  for  $2x$ , the length to be measured, and  $S$  for  $2s$ , the whole length of the tape,

$$L = \frac{S^2 - 4d^2}{4d} \log \frac{S+2d}{S-2d}, \quad (6)$$

the logarithm being Naperian.

If we develop the logarithm by the formula

$$\log \frac{1+x}{1-x} = 2x \left[ 1 + \frac{x^2}{3} + \frac{x^4}{5} + \dots \right],$$

we have

$$\begin{aligned} L &= S \left[ 1 - \frac{(2d)^2}{S^2} \right] \left[ 1 + \frac{(2d)^2}{3S^2} + \frac{(2d)^4}{5S^4} + \dots \right] \\ &= S \left[ 1 - \frac{2(2d)^2}{3S^2} - \frac{2(2d)^4}{3 \cdot 5S^4} - \frac{2(2d)^6}{5 \cdot 7S^6} - \frac{2(2d)^8}{7 \cdot 9S^8} - \dots \right]. \quad (7) \end{aligned}$$

The second term is of course the same that occurs in the case of a circular arc.

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### SOLUTIONS OF PROBLEMS IN NUMBER THREE.

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SOLUTIONS of problems in No. 3 have been received, as follows:

From Prof W. P. Casey, 259, 264; Alexander S. Christie, 261; Geo. Eastwood, 265; W. V. McKnight, 259; W. L. Marcy, 263; E. B. Seitz, 264, 265; Prof. J. Scheffer, 259 and answer to query at page 55. Also, Prof. J. H. Kershner and R. J. Adcock each sent ans. to query at p. 55.